Decentralized scheduling of sensor network for parameter estimation of distributed systems

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Abstract – *An approach to determine a scheduling policy for sensor network monitoring some spatial domain in order to identify unknown parameters of a distributed system is discussed. Given a finite number of possible sites at which sensors may reside the activation schedule for scanning sensors is provided so as to maximize a criterion defined on the Fisher information matrix associated with the estimated parameters. The original combinatorial problem is relaxed through operating on the density of sensors in lieu of individual sensor positions. As a result of adaptation of pairwise communication algorithms a numerical scheme is developed, which distributes the computational burden between the network nodes. The proposed approach is illustrated with simulations on a sensor network scheduling problem for a two-dimensional convective diffusion process.*

Keywords: distributed-parameter system, parameter estimation, scanning measurements, sensor network.

1 Introduction

The systems with spatio-temporal dynamics, commonly known as Distributed-Parameter Systems (DPSs) constitute one of the most general and important classes of systems which are widely used in modelling for a wide variety of real-world engineering problems. Recent developments in technical systems force engineers to search for more precise mathematical models of the considered phenomena, what leads directly to the description of the system at hand using partial differential equations (PDEs) as the lumped descriptions often become unsatisfactory and cannot provide a sufficient approximation of the investigated system. Despite the sophisticated formulation, such models provide a high quality and efficiency of simulations and control techniques.

One of the crucial problems encountered while trying to design an experimental set-up for parameter estimation of a distributed system is an appropriate configuration of the monitoring system in terms of proper selection of sensor locations. The impossibility to observe the system states over the entire spatial domain implies the question of where to locate discrete sensors and how to schedule the observations so as to accurately estimate the unknown system parameters. This question becomes especially important in the context of recent advances in distributed sensor networks [1–3].

Over the past years, laborious research on the development of strategies for efficient sensor placement has been conducted (for reviews, see papers [4–7] and comprehensive monographs [8,9]). However, most techniques communicated by various authors usually rely on exhaustive search over a predefined set of candidates and the combinatorial nature of the design problem is taken into account very occasionally [7]. Obviously, such an approach is feasible for a relatively small number of possible sensor locations, and becomes useless as the number of possible location candidates increases. Although the number of sensor placement techniques developed to manage the problems of practical scale is very limited (cf. [4,8,9]), some effective approaches have been proposed to cover various experimental settings, including stationary [6,10–12], scanning [13–15] or moving observations [5,16–23].

The main aim of this work is to substantially extend the efficient decentralized approach to sensor configuration reported in [24] to the more general setting of scanning sensor networks, where the observation system comprises multiple stationary sensors located at already specified locations and it is desired to activate only a subset of them during a given time interval while the other sensors remain dormant [25]. A reason for not using all the available sensors could be the reduction of the observation system complexity and/or the cost of operation and maintenance [7]. The basic idea is to adopt the results for the so-called clusterization-free designs set forth in [26]. These, in turn, are reminiscent of the idea of replication-free designs which have emerged relatively late in the context of spatial statistics (see the monographs [27, 28]). The delineated approach can be easily tailored to the framework of so-called *gossip* algorithms in which each node communicates with no more than one neighbor at each time instant [29, 30]. In effect, the resulting exchange-type algorithm for sensor scheduling is working in a fully decentralized way and is very easy to implement.

2 Optimal measurement strategy

2.1 System description

Consider a bounded spatial domain $\Omega \subset \mathbb{R}^d$ with sufficiently smooth boundary Γ, a bounded time interval $T = (0, t_f]$, and a distributed parameter system (DPS) whose scalar state y at a spatial point $x \in \bar{\Omega} \subset \mathbb{R}^d$ and time instant $t \in \bar{T}$ is governed by the partial differential equation (PDE)

$$\frac{\partial y}{\partial t} = \mathcal{F}\big(x, t, y, \theta\big) \quad \text{in } \Omega \times T, \tag{1}$$

where $\mathcal{F}$ is a well-posed, possibly nonlinear, differential operator which involves first- and second-order spatial derivatives and may include terms accounting for forcing inputs specified *a priori*, and θ being an m-dimensional vector of unknown constant parameters which must be estimated using observations of the system. The PDE (1) is accompanied by the appropriate boundary and initial conditions

$$\mathcal{B}(x, t, y, \theta) = 0 \qquad \text{on } \Gamma \times T, \tag{2}$$

$$y = y_0 \qquad \text{in } \Omega \times \{t = 0\}, \tag{3}$$

respectively, $\mathcal{B}$ being an operator acting on the boundary Γ and $y_0 = y_0(x)$ a given function. We assume that the forms of $\mathcal{F}$ and $\mathcal{B}$ are given explicitly up to vector θ.

Let assume that the state y is observed directly (an extension to more general situation where the state is observed indirectly can be provided without serious difficulties, see [9] for details) by N pointwise sensors, from among only n are activated at time instants $0 < t_0 < t_1 < \cdots < t_K = t_f$ and will gather the continuous measurements for the duration of each subinterval $T_k = (t_{k-1}, t_k]$, $k = 1, \ldots, K$. Forming such an arbitrary partition on the time interval T, the considered 'scanning' observation strategy can be formally represented as

$$\begin{aligned} z^\ell_{\mathrm{m}}(t) =& y(x^\ell_k, t; \theta) + \varepsilon(x^\ell_k, t), \quad t \in T_k, \\ & \ell = 1, \ldots, n, \quad k = 1, \ldots K \end{aligned} \tag{4}$$

where $z^\ell_{\mathrm{m}}(t)$ is the scalar output and $x^\ell_k \in X$ stands for the location of the ℓ-th sensor at time subinterval T_k, X signifies the part of the spatial domain Ω where the measurements can be made and $\varepsilon(x^\ell_k, t)$ denotes the measurement noise, which is customarily assumed to be zero-mean, Gaussian, spatial uncorrelated and white [31].

2.2 Estimation accuracy measure

Given the model (1)–(3) and the outcomes of the measurements $z^\ell_{\mathrm{m}}(\,\cdot\,)$, $\ell = 1, \ldots, n$ on time intervals T_k, estimate θ by $\widehat{\theta}$, a global minimizer of the output least-squares criterion

$$\mathcal{J}(\vartheta) = \sum_{k=1}^{K} \sum_{\ell=1}^{n} \int_T \big\{ z^\ell_{\mathrm{m}}(t) - y(x^\ell_k, t; \vartheta) \big\}^2 \, \mathrm{d}t \tag{5}$$

where $y(\,\cdot\,, \,\cdot\,; \vartheta)$ denotes the solution to (1)–(3) for a given value of the parameter vector ϑ.

Inevitably, the covariance matrix $\mathrm{cov}(\widehat{\theta})$ of the above least-squares estimator depends on the active sensor locations x^ℓ_k. This fact suggests that we may attempt to select them so as to yield best estimates of the system parameters. To form a basis for a comparison of different locations, a quantitative measure of the 'goodness' of particular sensor configurations is required. Such a measure Ψ is customarily based on the concept of the *Fisher Information Matrix* (FIM) which is widely used in optimum experimental design theory for lumped systems [27,32]. In our setting, owing to the character of noise in (4), the FIM is given by [31]

$$M = \sum_{k=1}^{K} \sum_{\ell=1}^{n} \frac{1}{t_f} \int_{T_k} g(x^\ell_k, t) g^{\mathsf{T}}(x^\ell_k, t) \, \mathrm{d}t, \tag{6}$$

where

$$g(x, t) = \left[\frac{\partial y(x, t; \vartheta)}{\partial \vartheta_1}, \quad \ldots, \quad \frac{\partial y(x, t; \vartheta)}{\partial \vartheta_m} \right]^{\mathsf{T}}_{\vartheta = \theta^0} \tag{7}$$

stands for the so-called *sensitivity vector*. Since in the nonlinear case g depend on the estimated parameters, some preliminary estimate θ^0 is required for its calculation. Usually some known nominal values of the parameters θ can be used or we can apply estimates obtained from previous experiments [8, 33, 34]. Up to a constant scalar multiplier, the inverse of the FIM constitutes a good approximation of $\mathrm{cov}(\widehat{\theta})$ provided that the time horizon is large, the nonlinearity of the model with respect to its parameters is mild, and the measurement errors are independently distributed and have small magnitudes [27].

As for a specific form of Ψ, various options exist [27,32], but the most popular criterion to be maximized, called the D-optimality criterion, is the log-determinant of the FIM:

$$\Psi(M) = \log \det(M). \tag{8}$$

The resulting D-optimum sensor configuration leads to the minimum volume of the uncertainty ellipsoid for the estimates.

2.3 Sensor scheduling problem

The optimal sensor scheduling problem consists in seeking for each time subinterval T_k the best subset of n locations from among the N given potential ones. More precisely, the problem is to divide for each time subinterval the N available sensor nodes into n active ones and the remaining $N - n$ dormant ones so as to maximize the criterion (8) associated with the parameters to be estimated. Introducing for each possible location x^i ($i = 1, \ldots, N$) a set of variables v^i_ks, each of them taking the value 1 or 0 depending on whether or not a sensor residing at x^i is activated during T_k, the FIM in (6) can then be rewritten as

$$M(v_1, \ldots, v_K) = \sum_{i=1}^{N} \sum_{k=1}^{K} v^i_k M_k(x^i), \tag{9}$$

where

$$M_k(x^i) = \frac{1}{t_f} \int_{T_k} g(x^i, t) g^{\mathsf{T}}(x^i, t)\, \mathrm{d}t \tag{10}$$

and $v^i = (v_1^i, \ldots, v_K^i)$. Hence, our design problem takes the following form:

Problem 1 *Find a sequence $v = (v^1, \ldots, v^N)$ to maximize*

$$\mathcal{P}(v) = \Psi\big(M(v)\big), \tag{11}$$

subject to

$$\sum_{i=1}^{N} v_k^i = n, \quad k = 1, \ldots, K, \tag{12}$$

$$v_k^i = 0 \text{ or } 1, \quad i = 1, \ldots, N, \quad k = 1, \ldots, K. \tag{13}$$

This constitutes a 0–1 integer programming problem which necessitates an original and efficient solution. In [9] and [8] a general sequential computational scheme was proposed to solve a similar problem based on the notion of so-called directly constrained design measures. Brief characterization of main ideas underlying such an approach and development of the appropriate decentralized numerical algorithm will constitute the remainder of the paper.

3 Approximate solution via continuous relaxation

3.1 Problem of optimal sensor densities

When the numbers of candidate sites and sensors to be activated n is large, we can operate on the spatial density of sensors (i.e., the number of sensors per unit area), rather than on the individual sensor locations. In order to get rid of the combinatorial nature of the original problem we relax the definition of the set of admissible solutions by making use of the observation that the density of sensors over the subinterval T_k can be approximately described by a probability measure $\xi_k(\mathrm{d}x)$ on the space $(X, \mathcal{B})$, where $\mathcal{B}$ is the σ-algebra of all Borel subsets of X. Such an extension of the set of feasible solutions makes it possible to apply convenient and efficient mathematical tools of convex programming theory. As for practical interpretation of the so produced results, e.g., is to partition X into non-overlapping subdomains X_i of relatively small areas and then, on each subinterval T_k, to allocate to each of them the number

$$N_r^i = \left\lceil N \int_{X_i} \xi_k(\mathrm{d}x) \right\rceil \tag{14}$$

of sensors (here $\lceil \rho \rceil$ is the smallest integer greater than or equal to ρ).

Accordingly, our aim is to find probability measures ξ_k, $k = 1, \ldots, K$ over X which are absolutely continuous with respect to the Lebesgue measure and satisfy by definition the conditions

$$\int_X \xi_k(\mathrm{d}x) = 1, \quad k = 1, \ldots, K. \tag{15}$$

The integration above is to be understood in the Lebesgue-Stieltjes sense. In what follows we briefly write $\xi = (\xi_1, \ldots, \xi_K)$ and call ξ a *design measure*.

Consequently, we replace (6) by

$$M(\xi) = \sum_{k=1}^{K} \int_X M_k(x^i)\, \xi(\mathrm{d}x). \tag{16}$$

This leads directly to the so-called *continuous* designs which constitute the basis of the modern theory of optimal experiments [8, 9, 27].

Furthermore, another natural assumption is that the densities of sensor allocation are bounded by some prescribed level, i.e.

$$\xi_k(\mathrm{d}x) \le \omega(\mathrm{d}x), \quad k = 1, \ldots, K \tag{17}$$

where $\omega(\mathrm{d}x)$ signifies the maximal possible 'number' of sensors per $\mathrm{d}x$ [8, 9, 27] such that $\int_X \omega(\mathrm{d}x) \ge 1$. Finally, we come up with the following optimization problem:

Problem 2 *Find a design measure $\xi \in \Xi(X)$, $\Xi(X)$ being the set of all probability measures on X, to maximize*

$$\mathcal{J}(\xi) = \Psi\big(M(\xi)\big) \tag{18}$$

subject to

$$\xi_k(\mathrm{d}x) \le \omega(\mathrm{d}x), \quad k = 1, \ldots, K. \tag{19}$$

The design $\xi^\star$ being the solution to the problem above is then said to be a (Ψ, ω)-*optimal design* [27].

3.2 Characterization of the optimal solutions

In order to derive an efficient decentralized algorithm for solving Problem 2, a number of important properties of the optimal design $\xi^\star$ can be provided from the general results in [8, 9]. In the remainder of this paper we shall make the following assumptions:

(A1) X is compact,

(A2) $g \in C(X \times T; \mathbb{R}^m)$,

(A3) $\widetilde{\Xi}(X) = \big\{\xi : \xi \text{ satisfy (19) and } J(\xi) \ne 0\big\} \ne \emptyset$,

(A4) $\omega(\mathrm{d}x)$ is atomless, i.e., for any $\Delta X \subset X$ there exists a $\Delta X' \subset \Delta X$ such that

$$\int_{\Delta X'} \omega(\mathrm{d}x) < \int_{\Delta X} \omega(\mathrm{d}x). \tag{20}$$

In what follows, we write $\bar{\Xi}(X)$ for the collection of all the design measures whose components satisfy

$$\xi_k(\Delta X) = \begin{cases} \omega(\Delta X) & \text{for } \Delta X \subset \operatorname{supp} \xi_k, \\ 0 & \text{for } \Delta X \subset X \setminus \operatorname{supp} \xi_k \end{cases} \tag{21}$$

where the support of a measure ξ is defined as the closed set $\operatorname{supp} \xi = X \setminus \bigcup\{G : \xi(G) = 0,\ G \text{ is open}\}$, cf. [35, p.80].

Given a design ξ, we will say that the function $\phi_k(\,\cdot\,,\xi)$ defined by

$$\phi_k(x,\xi) = \operatorname{trace}\left[M^{-1}(\xi)M_k(x)\right] \tag{22}$$

separates sets X_1 and X_2 with respect to $\omega(\mathrm{d}x)$ if for any two sets $\Delta X_1 \subset X_1$ and $\Delta X_2 \subset X_2$ with equal non-zero ω-measures we have

$$\int_{\Delta X_1} \phi_k(x,\xi)\,\omega(\mathrm{d}x) \geq \int_{\Delta X_2} \phi_k(x,\xi)\,\omega(\mathrm{d}x). \tag{23}$$

Now we are able to formulate the following property of (Ψ,ω)-optimal designs, see [8, 9] for details.

Theorem 1 *Let Assumptions (A1)–(A4) hold. Then:*

(i) There exists an optimal design $\xi^\star \in \bar{\Xi}(X)$, and

(ii) A necessary and sufficient condition for $\xi^\star = (\xi_1^\star,\ldots,\xi_K^\star) \in \bar{\Xi}(X) \cap \widetilde{\Xi}(X)$ to be (Ψ,ω)-optimal is that functions $\phi_k(\,\cdot\,,\xi^\star)$ separate $X_r^\star = \operatorname{supp}\xi_k^\star$ and its complement $X \setminus X_k^\star$ for $k = 1,\ldots,K$.

From a practical point of view, the above result means that at all the support points of an optimal design $\xi^\star$ the mapping $\phi_k(\,\cdot\,,\xi^\star)$ should be greater than anywhere else, i.e., preferably $\operatorname{supp}\xi_k^\star$ should coincide with maximum points of $\phi_k(\,\cdot\,,\xi^\star)$. In practice, this amounts to allocating observations to the points at which we know least of all about the system response.

4 Decentralized Exchange Algorithm

Clearly, unless the considered design problem is quite simple, we must employ a numerical algorithm to make the outlined idea useful. Moreover, the key property of the resulting procedure should be an effective distribution of computations between the sensor nodes in a fully decentralized way. In the following we assume the asynchronous time model for the configuration process. Let $r = 0, 1, 2, \ldots$ be the discrete time index, which partition the continuous configuration time axis into time slots $Z_r = (z_{r-1}, z_r]$.

Owing to Theorem 1, $\xi_k^\star(\mathrm{d}x)$ should be nonzero in the areas where $\phi_k(\,\cdot\,,\xi^\star)$ takes on a larger value. Thus the central idea when constructing a computational algorithm for sensor density optimization is to move some measure from areas with smaller values of $\phi_k(\,\cdot\,,\xi^{(r)})$ to those with larger values, as we expect that such a procedure will improve $\xi^{(r)}$. A fundamental question related to this issue is whether the functions $\phi_k(\,\cdot\,,\xi^{(r)})$ can be calculated or estimated in a decentralized way. From (22) we see that the only component of $\phi_k(\,\cdot\,,\xi^{(r)})$ which cannot be calculated independently of other nodes is the global information matrix $M(\xi^{(r)})$. Furthermore, from (9) it is clearly seen that this matrix is a weighted average of the local information matrices given by (10). In such a way, our task is closely related to the problem of distributed averaging on a sensor network which appears in many applications and has been a subject of extensive studies [29, 30, 36].

One of the simplest and popular techniques of distributed averaging is a pairwise communication flooding, also known as a *gossip* scheme, which in its classic version assumes that at the r-th time slot the i-th sensor contacts some neighboring node j with probability P_{ij}, i.e., a pair $(i-j)$ is randomly and independently selected. At this time, both nodes set their values equal to the average of their current values. Denoting as $M_\ell(\xi^{(r)})$ an estimate of global FIM maintained by ℓ-th sensor at time slot Z_k we have ($\leftarrow$ is an update operator)

$$M_\ell(\xi^{(r)}) \leftarrow \frac{1}{2}\Big(M_i(\xi^{(r)}) + M_j(\xi^{(r)})\Big), \quad \ell \in \{i,j\}. \tag{24}$$

However, in our setting the distributed averaging problem is not typical as not all of the nodes contribute to the global FIM at the r-th configuration slot. Therefore, apart from updating local estimates of the FIM the sensor nodes should be equipped with a mechanism to store and change the global design $\xi^{(r)}$. This can be achieved by the exchange of tokens representing the activation of the sensors at given subintervals T_k. Such tokens are transferred between nodes in the situation where a neighbor node at particular observation subinterval T_k is more informative in the sense of the function $\phi_k(\,\cdot\,,\xi^{(r)})$ calculated on the current estimates of FIM (24) (and, obviously, it is not activated yet).

The decided advantage of such an approach is that the structure of design $\xi^{(r)}$ is distributed via tokens over the whole network, therefore it is not required to record the structure of $\xi^{(r)}$ at each node. If the network node owns the tokens for some of subintervals T_k's, it is allowed to additionally contribute to its own estimate of FIM according to the following weighted average:

$$M_i(\xi^{(r)}) \leftarrow \frac{n-1}{n} M_i(\xi^{(r)}) + \frac{1}{n}\sum_{k=1}^{K} v_k^i M_k(x^i). \tag{25}$$

The first term enforces consensus among the nodes (represents the average information from the rest of network), while the second accounts for the increase in the total contribution of the active node at subinterval T_k, i.e. for $v_k^i = 1$. The idea of the communication process is embodied by algorithm 1.

The operator EXCHANGE involved in implementation of Algorithm 1 is responsible for duplex data exchange between two nodes, i.e., sending and receiving data to/from connected neighbor (order depending on who initiated communication). At $r = 0$ each network node starts with a global FIM estimate $M_i(\xi^{(0)})$ initialized with its local information matrix $M(x^i) = \sum_{k=1}^K M_k(x^i)$ and randomly allocated tokens satisfying (12). Then at each time slot Z_k an appropriate pair of nodes exchanges information according to Algortithm 1.

The resulting procedure may be considered as a distributed counterpart of the class of sequential exchange algorithms based on restricted design measures (cf. [8, 9]) originated from experimental design theory. In particular, in the framework considered, we have $\omega(\mathrm{d}x) = \varrho\,\mathrm{d}x$, where ϱ is a

Algorithm 1 Distributed data exchange model. Indexes i and j denote, respectively, data from local repository and obtained from neighbor.

procedure EXCHANGE_PROTOCOL
  EXCHANGE($M_j(\xi)$,$M_i(\xi)$) ▷ sends and receives FIM ...
  EXCHANGE(v^i,v^j) ▷ ... and status
  $M_i(\xi) \leftarrow \frac{1}{2}\big(M_i(\xi) + M_j(\xi)\big)$ ▷ FIM averaging
  for $k \leftarrow 1, K$ **do**
    if XOR(v_k^i, v_k^j) **then** ▷ the token may be exchanged
      $\phi_k(x^i, \xi) \leftarrow \mathrm{trace}\left[M_i^{-1}(\xi) M_k(x^i)\right]$
      EXCHANGE$\big(\phi_k(x^i, \xi), \phi_k(x^j, \xi)\big)$
      if $\phi_k(x^i, \xi) > \phi_k(x^j, \xi)$ **then**
        $v_k^i \leftarrow 1$ ▷ activation token goes to the
      else ▷ more informative node
        $v_k^i \leftarrow 0$
      end if
    end if
  end for
  $M_i(\xi) \leftarrow \frac{n-1}{n} M_i(\xi) + \frac{1}{n} \sum_{k=1}^{K} v_k^i M_k(x^i)$.
end procedure

sensor density and every grid element must not contain more than one supporting point.

As regard the convergence to an optimal design, it is assured only for decreasing value of exchanged design measure, which is not the case here. Therefore, some oscillations in $\mathcal{J}(\xi^{(r)})$ may sometimes be observed. A denser spatial grid usually constitutes a remedy for this predicament [8, 9, 28].

Another important issue is the choice of proper communication scheme which significantly influence the convergence rate. In general, under some assumptions on the network connectivity graph, a suitable gossip algorithm can be provided with an exponential convergence rate. Since discussion on such important issues is far beyond the scope of this work, therefore we refer the reader to the seminal papers [29, 30] for details.

5 Simulation example

As an illustrative example of the presented approach, consider the problem of sensor configuration for parameter estimation in the process of air pollutant transport over a given urban area Ω, being a square with a side of length 1 km. In this domain, two active sources of pollution are present, which yields the pollutant spatial concentration $y = y(x, t)$. The evolution of y over the observation interval $T = (0, 1000]$ (in seconds) is described by the following advection-diffusion-reaction equation:

$$\begin{aligned} \frac{\partial y(x,t)}{\partial t} + \nabla \cdot \big(v(x,t) y(x,t)\big) + \alpha y(x,t) \\ = \nabla \cdot \big(\kappa \nabla y(x,t)\big) + f_1(x) + f_2(x), \quad x \in \Omega \end{aligned} \tag{26}$$

subject to the boundary and initial conditions:

$$\frac{\partial y(x,t)}{\partial n} = 0, \quad \text{on } \partial\Omega \times T, \tag{27}$$

$$y(x, 0) = y_0, \quad \text{in } \Omega, \tag{28}$$

where terms $f_\ell(x) = \mu_\ell \exp\big(-100\|x - \xi^\ell\|^2\big)$, $\ell = 1, 2$ represent the pollutant sources with emission intensities μ_ℓ located at the points $\xi^\ell = (\xi_1^\ell, \xi_2^\ell)$, $\ell = 1, 2$, and $\partial y/\partial n$ stands for the partial derivative of y with respect to the outward normal to the boundary $\partial\Omega$. The average spatio-temporal changes of the wind velocity field over Ω were approximated according to the model (scaled in [km/h])

$$v(x,t) = 7.2 \cdot \Big((x_1 + x_2 - t \cdot 10^{-3}), (2x_1 - 1)t \cdot 10^{-3} + x_2 - 1\Big).$$

Furthermore, κ denotes an unknown turbulent diffusion coefficient and $\alpha = 0.01$ s^{-1} stands for the absorption rate modelling a slow decay of the pollutant. Figure 1 illustrates the resulting complex process dynamics.

Our goal is to identify the sources and unknown diffusion coefficient, i.e., an estimate the vector $\theta = (\mu_1, \xi_1^1, \xi_2^1, \mu_2, \xi_1^2, \xi_2^2, \kappa)$ using a sensor network with scanning nodes. The observation horizon was split into 10 evenly partitioned subintervals $T_k = (100(k-1), 100k]$, $k = 1, \ldots, 10$. In order to verify the proposed approach, a MATLAB program was written using a PC equipped with Intel Centrino T9300 processor (2.5GHz, 3 GB RAM) running Windows Vista and MATLAB 2007b. First, the system of PDEs was solved using efficient solvers of the COMSOL environment based on the finite element method [37]. The nominal values of the system parameters were assumed to be $\theta = (10\mathrm{kg/s}, 0.5\mathrm{km}, 0.75\mathrm{km}, 15\mathrm{kg/s}, 0.75\mathrm{km}, 0.5\mathrm{km}, 50\mathrm{m}^2/\mathrm{s})$. Calculations were performed for a spatial mesh composed of 978 triangles, 520 nodes and an evenly partitioned time interval (101 subintervals).

The observation grid was assumed to be created at locations selected from among those elements of above-mentioned 520-point triangulation mesh which do not lie on the outer boundary (there were 460 such nodes, which are indicated with dots in Fig. 2). Given $N = 460$ prospective sites in Ω, we aim at selecting at each time subinterval T_k their subset consisting of the locations at which the measurements made by $n = 150$ sensors would lead to D-optimum least-squares estimates of the parameters θ.

It was assumed that the network is fully connected with uniform probability distribution for the connection between selected pair of nodes. The complexity of the system dynamics makes the proper prediction of the observation locations rather difficult and nonintuitive. The sensors tend to form patterns reflecting the areas of greatest changes in the pollutant concentration. Sensor configurations at different stages of algorithm are shown in Fig. 2. The convergence is very fast at the first stage what is clearly seen in Figs. 2. After a relatively small configuration time r a sensor activation pattern is very similar to the optimal one (cf. Figs. 2(c) and (d)). Then the algorithm considerably slows down and convergence with high accuracy require many pairwise communications. An application of a better communication scheme would be helpful here to overcome this effect.

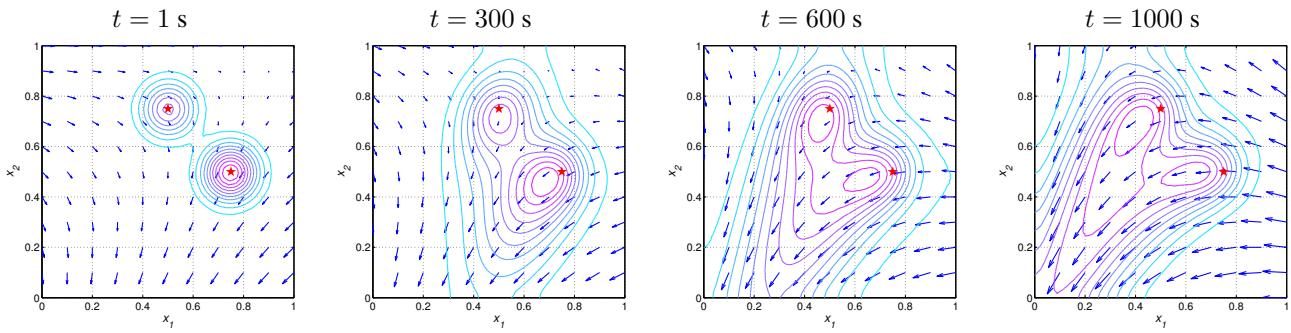

Figure 1: Temporal changes in the wind velocity field and pollutant concentration.

6 Conclusion

The sensor scheduling problem in view of accurate parameter estimation for distributed-parameter systems subject to limitations on the total number of activated sensor nodes has been addressed. The main contribution of this work is a proper characterization of an approach based on the conversion of the original combinatorial optimization problem to its relaxed continuous approximation which is further used to provide a decentralized scheme for computation and information exchange. The presented results show that some methods of optimum experimental design can be extended to the considered setting of scanning observations without complex modifications. As a result, an efficient and fully distributed algorithm is developed which is capable of solving scheduling tasks for large-scale sensor networks. Extensions to more sophisticated monitoring systems, i.e., mobile sensor networks constitute future research directions.

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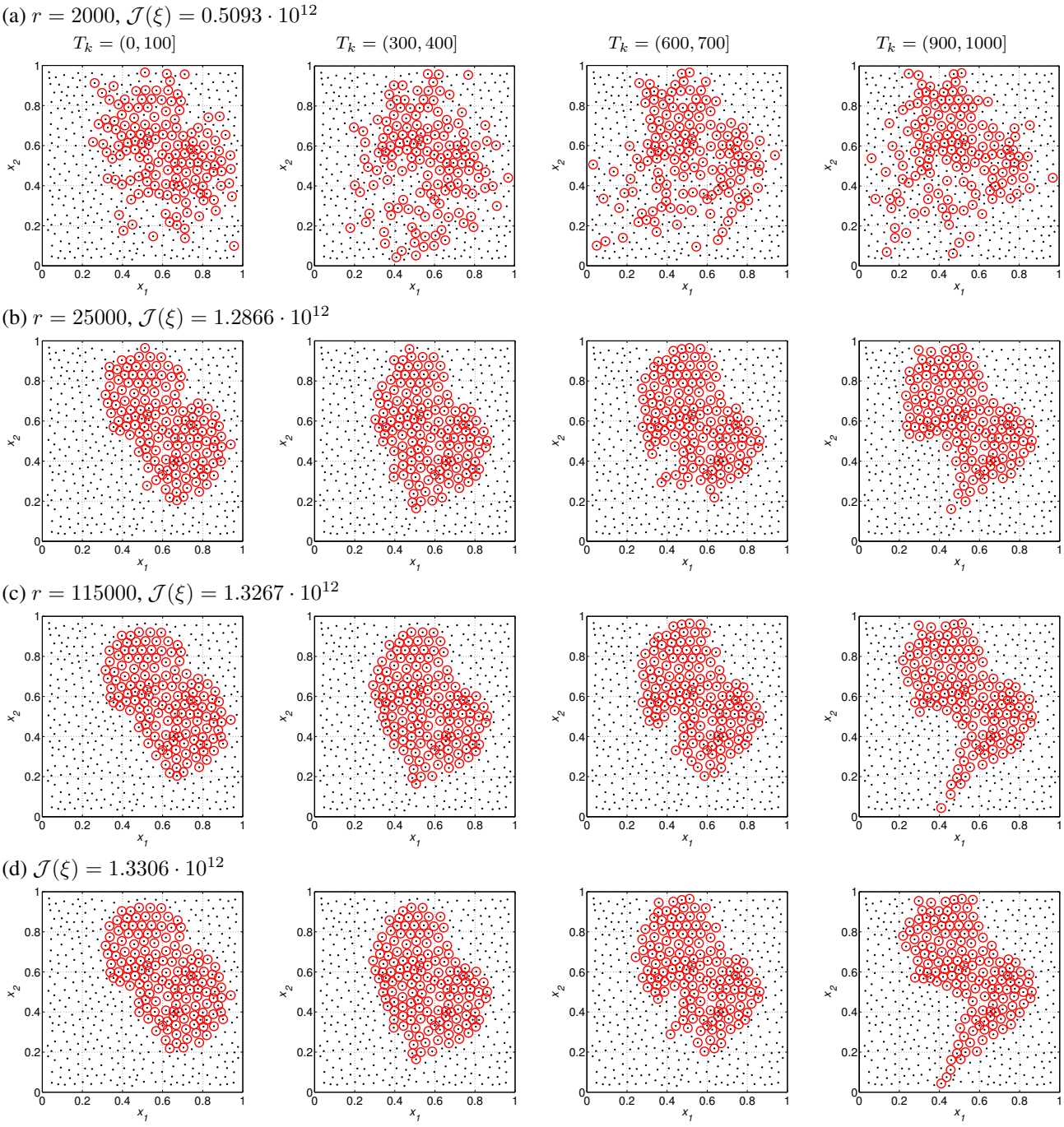

Figure 2: Allocation of active sensors at chosen observation subintervals in consecutive stages of network configuration (a)-(c) and final D-optimal configuration (d).

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